International Trade and the Interaction of Labour Market Frictions and Endogenous Firms Exit: An Examination of Labour Productivity and Trade Dynamics

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Abstract

How do the interactions between non-trading firms and labour market frictions impact the domestic and international transmission of macroeconomic shocks? In this paper we explore the impact of these interactions on labour productivity and unemployment, in a model with endogenous fluctuations in the entry and exit of non-trading firms into and out of the domestic market and labour market frictions. We find that in such a model, these interactions generate larger fluctuations in the persistence of labour productivity than models with only endogenous entry and exit. Our model also offer a framework in which we can examine the drivers of the wide variation in empirically calculated figures for the persistence of productivity.

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1 Introduction

Key recent paper show that labour market rigidities contribute to shaping the pattern of international trade. However, despite major contributions, the joint impact of labour market rigidities and the entry and exit of less-productive non-trading firms into and out of the domestic market has not been yet studies in this literature. In this paper, we study the impact of the interactions between labour market frictions and the entry and exit of less-productive non-trading firms into and out of the domestic market on the response of labour productivity and unemployment to macroeconomic shocks. The analysis takes place in a dynamic, stochastic, general equilibrium (DSGE) open economy model of international trade with monopolistic competition, heterogeneous firms and labour market frictions.

The impact of the entry and exit of trading firms into and out of exporting markets has been extensively examined in the recent literature, for example by Ghironi and Melitz (2005), Bekes and Murakozy (2012), Impullitti *et al.* (2013) and Fattel Jaef and Lopez (2014). These papers show that the dynamic entry and exit response of trading firms impacts the persistence of exporting, and the persistence of exporting further impacts the persistence of consumption (Ghironi and Melitz (2005)) and TFP (Fattel Jaef and Lopez (2014)). More recently, research has extended the existing analysis to account for the behaviour of non-trading firms. Millard *et al.* (2019) extend the framework in Ghironi and Melitz (2005) to allow for the endogenous entry and exit of non-trading firms into and out of the domestic market. They show that the entry and exit of non-trading firms introduces endogenous persistence into the response of productivity to macroeconomic shocks, that varies with the origin of the shock, with technology shocks giving a more persistent productivity response than shocks to the sunk costs of entering the market, which in turn give a more persistent productivity response than shocks to the fixed costs of producing for the domestic market. Important theoretical precursors to the analytical foundations to these papers are contained in Dixit and Stiglitz (1977), Krugman (1979), Krugman (1980), Hopenhayn (1992*a*), Hopenhayn (1992*b*) and Melitz (2003).

The impact of the interactions between labour market frictions and the entry and exist of trading firms into and out of exporting markets on the transmission of macroeconomic shocks and policy reforms has also been extensively examined, see for example, Cacciatore (2014), Cacciatore, Duval, Fiori and Ghironi (2016*a*), Cacciatore, Duval, Fiori and Ghironi (2016*b*), Cacciatore, Fiori and Ghironi (2016), Cacciatore and Fiori (2016). These papers show that labour market frictions, and the resultant flexibility or rigidity of labour markets play an important role in driving the transmission of macroeconomic shocks internationally and domestically, as well as an important role in transmitting the effects of policy reforms domestically and internationally. However, this literature only allows for interactions between the entry and exist of trading firms into and out of exporting markets and labour market frictions, while the interactions between the entry and exit of non-trading firms into and out of the domestic market are abstracted from, as the productivity of domestic production is exogenously fixed.

This paper also builds on the literature examining solely the impact of firm entry and exit on productivity, which started with Hopenhayn (1992*a*) and Hopenhayn (1992*b*). Key theoretical contributions are by Bilbie *et al.* (2012), Clementi and Palazzo (2016), Woo (2016), Hamano and Zanetti (2017) and Lee and Mukoyama (2018) while Moreira (2017) and Sedlacek and Stern (2017) provide an empirical analysis. Our paper builds on this literature by providing a framework which allows for the analysis of the interactions between firms entry and exit and labour market frictions, in contrast to solely the impact of firm entry and exit.

This paper is also closely related to the literature examining the impact of labour market frictions on the entry and exit of firms and vice versa in a closed economy. The main theoretical contributions are Shao and Silos (2013), Colciago and Rossi (2015*a*), Colciago and Rossi (2015*b*), and Kaas and Kimasa (2018), while Colciago *et al.* (2019) examine the link between firm entry and exit and job flows empirically. Our paper builds on this literature by firstly extending their closed economy analysis to an open economy framework and secondly examining the impact of the interactions between firm entry and exit and labour market frictions. Studying the impact of these interactions in an open economy context allows undertaking an analysis of the overall effect of international trade flows on the interactions between firm entry and exit and labour market frictions, and the labour productivity impact on the transmission of macroeconomic shocks.

Our paper builds on these streams of literature by building a model with both: endogenous entry and exit of non-trading firms into and out of the domestic market in an open economy setup and labour market frictions modelled in the manner of Pissarides (1985), Mortensen and Pissarides (1994) and Pissarides (2001). Given the importance of firm entry and exit in driving job dynamics (see Burgess *et al.* (2000) and Haltiwanger (2011)), these interactions are likely to play a key role in the transmission of macroeconomic shocks through labour markets and the rest of the economy. Using our model, we show that the interactions between the endogenous entry and exit of non-trading domestic firms into and out of the domestic market, and the labour market frictions generates further endogeneity into the persistence of productivity, above and beyond the persistence generated solely by non-trading firm entry and exit, as in Millard *et al.* (2019), showing that labour market rigidities do affect productivity through their impact on the less productive non-trading firms (affected more by the hiring and firing cost than the more productive firms) and making labour productivity more persistent.

The rest of the paper is organised as follows: Section 2 presents the theoretical model. Section 3 presents the calibration of the model. Section 4 presents and discusses the interactions between the entry and exit of non-trading firms into and out of the domestic market and labour market frictions, and the impact of these interactions on labour productivity, unemployment and other main macro variables in the transmission of macroeconomic shocks. Section 5 concludes.

2 Model

Our framework builds on the model of Millard *et al.* (2019), itself an extension of Ghironi and Melitz (2005), in which we allow for a labour market which is characterised by search and matching frictions as in Mortensen and Pissarides (1994), as modelled by Cacciatore (2014). Thus, we develop a two-country, home and foreign, model with labour market frictions and endogenous entry and exit, driven by endogenous average firm-level productivity. In our exposition of the model, we focus on the Home economy, with the understanding that analogous equations hold for the Foreign country. Foreign variables are denoted with a superscript star.

2.1 Households

Households are homogeneous and demand goods from both home and foreign producers. The representative household in Home country supplies L_t units of labour to the producers in Home country at an average nominal wage rate $\tilde{W}_t$; the average real wage rate is denoted by $\tilde{w}_t$. Households accumulate risk free foreign bonds and shares in the domestic firms, and issue risk free domestic bonds. The representative household maximises the expected intertemporal utility from consumption, subject to their budget constraint:

$$\max_{w.r.t:L_t,C_t,B_{t+1},B^*_{t+1},x_{t+1}} E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\frac{C_s^{1-\gamma}-1}{1-\gamma}-\upsilon L_t^S\right], \tag{1}$$

$$\begin{aligned}(1+r_t)B_t+\xi(B_{t+1})^2/2+Q_tB^*_{t+1}+\xi Q_t(B^*_{t+1})^2/2+\tilde{vl}_tN_t^Hx_{t+1}+C_t=&B_{t+1}+\\ +Q_t(1+r^*_t)B^*_t+(\tilde{d}_t+\tilde{vl}_t)N_t^Dx_t+\tilde{w}_tL_t+u^b(L^F-L_t)+T_t^B-T_t^U&,\end{aligned}$$

where $\beta\in(0,1)$ is the consumer discount factor, $\gamma>0$ is the inverse of the intertemporal elasticity of substitution, υ is the disutility from work, [1] L_t^S is the quantity of labour supplied by the household and C_t is the consumption basket which aggregates imported and domestic goods in every period. C_t is defined over a continuum of goods, $C_t=\left(\int_{\omega\in\Omega_t}c_t(\omega)^{\frac{\theta-1}{\theta}}d\omega\right)^{\frac{\theta}{\theta-1}}$, where $\theta>1$ is the elasticity of substitution across goods. B_t is the household's issue of domestic bonds at the beginning of time t, on which they pay a risk free rate of interest r_t, and B^*_t is the household's holdings of foreign bonds at the beginning of period t, which pay a risk free rate of interest r^*_t. Both the domestic and foreign interest rates are predetermined in period t.

To ensure stationary, we follow Turnovsky (1985) and Cacciatore (2014) and assume that households face a quadratic cost of adjusting bond holdings/issuance: $\xi(B_{t+1})^2/2$, and $\xi(B^*_{t+1})^2/2$, where $\xi>0$ is a scalar. L^F is the labour force, u^b is unemployment benefit, T_t^B is a lump sum rebate of the cost of adjusting bond holdings, and T_t^U are lump sum taxes from the government at time t. In equilibrium, $T_t^B=\xi(B_{t+1})^2/2+\xi Q_t(B^*_{t+1})^2/2$ and $T_t^U=u^b(L^F-L_t^S)$. Q_t is the real exchange rates between the Home and the Foreign country, denoting units of Home consumption good per units of Foreign consumption good. x_t are the representative household's holdings of shares in a mutual fund of home firms, at the beginning of a period, chosen during the previous period, $\tilde{vl}_t$ and $\tilde{d}_t$ are the average value and per period profits of firms respectively. N_{Dt} is the number of firms that produce during a period and N_{Ht} is the total number of firms at the end of the period. At the beginning of a period, after shocks hit, but before production commences, firms choose whether to produce in that period, or exit the market. Thus, the number of firms that produce during a period, N_{Dt}, will be equal to the number of firms operating in the market at the end of the previous period, N_{Ht-1}, adjusted to reflect the proportion of firms that die out. The derivation of the values of $\tilde{vl}_t$, $\tilde{d}_t$, N_t^D and N_t^H will be presented in the next sections.

In each period, only a subset of goods, $\Omega_t\in\Omega$, will be available to consumers. Let $p_t(\omega)$ denote the nominal price of a good $\omega\in\Omega_t$. The consumption based price index is $P_t=\left(\int_{\omega\in\Omega_t}p_t(\omega)^{1-\theta}d\omega\right)^{\frac{1}{1-\theta}}$ and the consumer demand curve is given by $c_t(\omega)=\left(\frac{p_t(\omega)}{P_t}\right)^{-\theta}C_t$, for each individual good ω. Solving the households maximisation problem yields the optimal labour supply equation, as well as the Euler equations for bond and share holdings/issuance.

2.2 Firms

There is a continuum of monopolistically competitive firms in each of the two countries Home and Foreign, each producing a different variety of good $\omega\in\Omega$, which can be sold domestically and abroad. Firm ω employs $l_t(\omega)$ units of labour. Firm output depends on: (i) an aggregate technology level, Z_t, which evolves according to an AR(1) process with persistence ρ, which is common to all firms within a country, but allowed to vary between countries; (ii) an idiosyncratic firm-specific productivity, z; and (iii) an idiosyncratic job-specific productivity, $a_{z,t}$, for each relationship between a worker and firm. As in Millard *et al.* (2019), the idiosyncratic firm specific productivity is drawn by the firm from a distribution

[1] If $\gamma=1$, then U_t=ln$(C_t)-\upsilon L_t^S$.

$G(z)$ with support on $[z_{min},\infty)$, at the end of every period. The idiosyncratic job-specific productivity is drawn each period from a distribution with c.d.f $H(a)$ with support $(0,\infty)$, as in Cacciatore (2014), and the realisation of the job-specific productivity shock does not vary with the firm level productivity, z. Assuming that these two productivities are drawn every period in our model, ensures that the Melitz (2003) and Ghironi and Melitz (2005) proposition that average productivities are a fixed proportion of the cutoff productivity, as in Melitz (2003) and Ghironi and Melitz (2005), holds even in the presence of fixed costs of domestic production (in the case of firm level productivity) and labour market frictions (in the case of job specific productivity). This assumption means that the firm-level productivity of an individual firm will have zero persistence. Although this is a strong assumption, and means that many of the persistence properties of the real world are absent, it is necessary to ensure model tractability.

Given that each firm produces a single variety of good, ω, and that the firms optimal behaviour is determined by their firm-level productivity level z, we move from indexing by ω to indexing by z, such that $c_t(\omega) \equiv c_t(z)$ and $p_t(\omega) \equiv p_t(z)$ for a firm with a given productivity z. Thus, the total output of a firm with productivity level z is given by:

$$y_t(z) = zZ_t\tilde{a}_{z,t}l_t(z),$$

where $\tilde{a}_{z,t} \equiv (1 - H(a^c_{z,t}))^{-1} \int_{a_{z,t}}^{\infty} adH(a)$, and $a^c_{z,t}$ is an endogenously determined cutoff level of job specific productivity, below which the cost of retaining the job is greater than the cost of termination for the firm, given by the real cost of firing F.

To enter the market, and then draw a firm-level productivity for production in the following period, a firm must pay a sunk entry cost, f^E, denominated in units of the aggregate consumption good, as in Hopenhayn (1992*b*) and Melitz (2003), as well as hiring sufficient workers. In the manner of Hopenhayn (1992*a*) and Melitz (2003), but unlike in the models in Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014), firms also have to pay a per-period fixed cost of production, f^D, denominated in units of the aggregate consumption good. Any exporting firms will also pay a per-period costs of selling to the Foreign market, f^X, also denominated in units of the aggregate consumption good. In addition, exporting firms have to pay a per-unit iceberg cost, such that a firm needs to export τ units of their good in order to sell one unit in the destination market. Finally, we assume that the markets are monopolistically competitive. Each firm will continue to produce until, given the presence of the fixed costs of domestic production, they draw a productivity at the end of the previous period that is not high enough to make profits in the domestic market, in which case the firm will choose to exit the market. The firms' problem is to maximise the discounted stream of profits subject to their production function and the two consumer demand curves.

When employing labour, the process of job creation is subject to matching frictions, in the style of Pissarides (1985), Mortensen and Pissarides (1994) and Pissarides (2001). In order to post a vacancy, a firm must incur a real cost κ, expressed in units of the final consumption basket. The probability that the posted vacancy will result in a match for the firm depends on a constant returns matching function, which converts aggregate vacancies, V_t, and aggregate unemployed workers, U_t, into aggregate matches: $M_t = \chi {U_t}^{\varepsilon} {V_t}^{1-\varepsilon}$, with $0 < \varepsilon < 1$, and χ is the matching efficiency, $0 < \chi < 1$. Note that at the time of hiring and firing in period t, the aggregate unemployment is equal to the number of workers unemployed at the end of the previous period, plus the number of workers employed by firms that endogenously exit the market, and a fraction of jobs λ^x, which are exogenously separated, at no cost to the firm. The probability of a vacancy posted by a firm resulting in a match is therefore given by $q_t = M_t/V_t = \chi(U_t/V_t)^{\varepsilon}$, and the probability that an unemployed worker will meet a match is given by $\iota_t = M_t/U_t = \chi(U_t/V_t)^{\varepsilon-1}$. Therefore, for an individual firm, the number of new hires in a period will be equal to $q_t v_{z,t}$, where $v_{z,t}$ is the number of vacancies posted by a firm with productivity z at time t.

The timing of hiring and firing for a particular firm is as follows: at the end of the period, each firm draws their idiosyncratic productivity z for the following period. At the beginning of the next period, the exogenous job separation shock hits, and a fraction, λ^x, of the firms workers are separated at no cost. All aggregate shocks then hit, after which the firm decide whether or not to remain in the market, on the basis of their firm level productivity. If the firm exits the market, all the labour employed by that firm is separated. Once these firms have exited the market, the remaining firms posts vacancies. Once the workers are hired, the idiosyncratic job specific productivity shock hits, and all workers that draw a productivity lower than $a^c_{z,t}$ are fired. After these workers have been fired, all remaining workers produce during the period. As in Cacciatore (2014), the law of motion for employment within firms with productivity z is therefore given by:

$$l_t(z) = (1 - \lambda_{z,t})[(1 - \lambda^x)l_{t-1}(z) + q_t v_t(z)], \tag{2}$$

where $\lambda_{z,t} \equiv H(a^c_{z,t})$, is the endogenous separation rate. An incumbent firm with productivity z sets the price, production, number of vacancies, cutoff job specific productivity and labour employed, in order to maximise the discounted expected future profit stream. Given that the firms are entirely owned by the households, the discount rate will be given by $\beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$. The problem for a firm with idiosyncratic productivity z is to maximise its present discounted value:

$$\begin{aligned} \text{Max: } d_t(z) = E_t \Bigg[\sum_{s=t}^{\infty} \beta_{t,s}(1 - G(z_{Ds}))^{s-t} \big[p^D_s(z) y^D_s(z) + p^X_s(z) y^X_s(z) \epsilon_t \\ -\tilde{W}_s(z) l_s(z) - f^D - f^X - \kappa v_t(z) - \lambda_{z,s} F((1 - \lambda^x) l_{t-1}(z) + q_s v_s(z)) \big] \Bigg], \end{aligned} \tag{3}$$

$$w.r.t : p^D_t(z), y^D_t(z), p^X_t(z), y^X_t(z), v_t(z), a^c_{z,t}, l_t(z),$$

$$s.t : y^D_t(z) + \tau y^X_t(z) = z Z_t \tilde{a}_{z,t} l_t(z), \tag{4}$$

$$l_t(z) = (1 - \lambda_{z,t})[(1 - \lambda^x)l_{t-1}(z) + q_t v_t(z)], \tag{5}$$

$$y^D_t(z) = \left(\frac{p^D_t(z)}{P_t}\right)^{-\theta} Y^C_t, \tag{6}$$

$$y^X_t(z) = \left(\frac{p^X_t(z)}{P^*_t}\right)^{-\theta} Y^{C^*}_t, \tag{7}$$

where, $\beta_{t,s} = \beta E_t \left(\frac{C_s}{C_t}\right)^{-\gamma}$, τ is the iceberg costs for exports from the Home country to the Foreign country, $p^D_t(z)$, and $p^X_t(z)$ are the price of domestic goods and exports to the Foreign country denominated in units of Home and Foreign currency respectively, $y^D_t(z)$ and $y^X_t(z)$ are the total units of goods sold by the firm in the domestic market and exports to the Foreign country (it is assumed that supply matches demand, so $y_t(z) = c_t(z)$); $l_t(z)$ is the amount of labour used in production, $G(z_{Dt})$ is the probability that a firm draws a productivity below the cutoff level for domestic production and thus exits the market in period t, C_t and C^*_t are aggregate consumption in the Home and Foreign countries respectively, P_t and P^*_t are the consumption-based price indices in the Home and Foreign countries and, ϵ is the nominal exchange rate (units of Home currency per unit of Foreign currency) for Home country with the Foreign country. Finally, $\tilde{W}_{z,s} \equiv (1-H(a^c_{z,t}))^{-1}\int_{a^c_{z,t}}^{\infty} w_{z,t}(a)dH(a)$ is the average wage paid by the firm z, weighted according to the distribution of job specific productivities. As in Cacciatore (2014), wages are not identical across workers, but they depend on the idiosyncratic job-specific productivity, $a_{z,t}$.

Solving the firms problem, yields the following first order conditions for labour, vacancies and the cutoff job specific productivity level, in real terms:

$$\frac{\partial \mathcal{L}}{\partial l_t(z)} = -\tilde{w}_{z,s} + \varphi_{z,t} z Z_t \tilde{a}_{z,t} - \alpha_{z,t} - \beta_{t,t+1}(1-G(z_{Dt+1}))\lambda_{z,t+1}F(1-\lambda^x) + \\ +\alpha_{z,t+1}\beta_{t,t+1}(1-G(z_{Dt+1}))(1-\lambda^x)(1-\lambda_{z,t+1}) = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial v_t(z)} = -\kappa - \lambda_{z,t}Fq_t + \alpha_{z,t}(1-\lambda_{z,t})q_t = 0$$

$$\alpha_{z,t} = \frac{\kappa + \lambda_{z,t}Fq_t}{(1-\lambda_{z,t})q_t} \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial a^c_{z,t}} = -\frac{\partial \tilde{w}_{z,s}}{\partial a^c_{z,t}} l_t(z) - \frac{\partial \lambda_{z,t}}{\partial a^c_{z,t}} F((1-\lambda^x)l_{t-1}(z) + q_t v_{z,t}) + \varphi_{z,t} z Z_t l_{z,t} \frac{\partial \tilde{a}_{z,t}}{\partial a^c_{z,t}} \\ -\alpha_{z,t}\frac{\partial \lambda_{z,t}}{\partial a^c_{z,t}}((1-\lambda^x)l_{t-1}(z) + q_t v_{z,t}) = 0$$

$$\frac{\partial \tilde{w}_{z,s}}{\partial a^c_{z,t}} - \frac{\partial \lambda_{z,t}}{\partial a^c_{z,t}}\frac{F}{1-\lambda_{z,t}} + \varphi_{z,t} z Z_t \frac{\partial \tilde{a}_{z,t}}{\partial a^c_{z,t}} - \alpha_{z,t}\frac{\partial \lambda_{z,t}}{\partial a^c_{z,t}}\frac{1}{1-\lambda_{z,t}} = 0, \tag{10}$$

where $\varphi_{z,t}$ is the Lagrange multiplier on the firms production function (4), and corresponds to the real marginal cost of the firm and $\alpha_{z,t}$ is the Lagrange multiplier on the labour law of motion (5). Combining (8) and (9), the following job creation equation is obtained:

$$\frac{\kappa}{q_t} = \left[\varphi_{z,t} z Z_t \tilde{a}_{z,t} - \tilde{w}_{z,s} + (1-\lambda^x)E_t\left[\beta_{t,t+1}(1-G(z_{Dt+1}))\frac{\kappa}{q_{t+1}}\right]\right](1-\lambda_{z,t}) - \lambda_{z,t}F, \tag{11}$$

where the marginal cost and the marginal benefit of filling a vacancy are equalised. Combining the job creation equation with the first order condition for the cutoff job specific productivity level (10), the following job destruction equation is obtained:

$$\varphi_{z,t} z Z_t a^c_{z,t} - w_{z,t}(a^c_{z,t}) + (1-\lambda^x)E_t\left[\beta_{t,t+1}(1-G(z_{Dt+1}))\frac{\kappa}{q_{t+1}}\right] = -F. \tag{12}$$

At the optimum, the value to the firm of a job with productivity $a^c_{z,t}$ must be equal to zero, implying that the contribution of the match to current and expected future profits is exactly equal to the firm's outside option, firing the worker, paying F.

Wage Determination

The wage paid by the firm, to a worker with job specific productivity a, is determined by the following sharing rule:

$$\eta S^F_{z,t}(a) = (1-\eta)S^W_{z,t}(a), \tag{13}$$

where η is the Nash bargaining power of the worker, $S^F_{z,t}(a)$ is the firm's surplus and $S^W_{z,t}(a)$ is the worker's surplus. The firm's surplus will be equal to the value of the job to the firm, $\Gamma_{z,t}(a)$, plus a saving from not having to pay the firing cost F. The value of the job to the firm is given by the marginal value product of the match, plus the expected future value of continuation, minus the wage bill:

$$\Gamma_{z,t}(a) \equiv \varphi_{z,t} z Z_t a_t - w_{z,t}(a) + (1-\lambda^x) E_t \left[\beta_{t,t+1}(1-G(z_{Dt+1})) \left[(1-\lambda_{z,t+1})\tilde{\Gamma}_{z,t+1} - \lambda_{z,t+1} F\right]\right],$$

where $\tilde{\Gamma}_{z,t+1}$ corresponds to $\alpha_{z,t+1}$. The worker's surplus meanwhile is given by the current wage, minus the workers outside option, plus the expected future surplus from the match:

$$S^W_{z,t}(a) \equiv w_{z,t}(a) - \varpi + (1-\lambda^x) E_t \beta_{t,t+1}(1-G(z_{Dt+1}))(1-\lambda_{z,t+1})\tilde{S}^W_{z,t},$$

where $\tilde{S}^W_{z,t}$ is the average worker surplus at the firm with productivity z, $\varpi = \upsilon(C_t)^\gamma + u^m + (1-\lambda^x) E_t \beta_{t,t+1}(1-G(z_{Dt+1}))\iota^h_{t+1}\tilde{S}^W_{t+1}$ and $\tilde{S}^W_t$ is the average worker surplus at all domestic firms.

Solving for the average wage, as in Cacciatore (2014), it can be shown that all firms set the same cutoff productivity level and pay the same wage, irrespective of their productivity z. It can also be shown that the difference between the wage paid to the average worker and the worker with cutoff productivity is given by $\tilde{w}_{z,t} - w_{z,t}(a^c_{z,t}) = \eta \varphi_{z,t} z Z_t(\tilde{a}_{z,t} - a^c_{z,t})$. Thus, the worker's outside option is given by:

$$\varpi = \upsilon(C_t)^\gamma + u^m + \eta/(1-\eta) E_t \left[(1-\lambda^x)\beta_{t,t+1}(1-G(z_{Dt+1}))(\kappa\zeta_{t+1} + \iota_{t+1}F)\right],$$

where $\zeta_t = \iota_t / q^h_t$ represents the tightness of the labour market. From the expression for the workers outside option, the final expression for the average wage is obtained:

$$\begin{aligned} \tilde{w}_{z,t} = \eta\,&(\varphi_t Z_t \tilde{a}_{z,t} + \kappa E_t \left[(1-\lambda^x)\beta_{t,t+1}(1-G(z_{Dt+1}))\zeta_{t+1}\right] \\ &+ \left[1 - E_t\left[(1-\lambda^x)\beta_{t,t+1}(1-G(z_{Dt+1}))(1-\iota_{t+1})\right]F\right]) + (1-\eta)(\upsilon(C_t)^\gamma + u^m), \end{aligned} \tag{14}$$

where $\varphi_t = \varphi_{z,t}/z$ is an expression for the average marginal cost.

Firm Profits

Solving the original firm's problem, also yields the result that firms set their output price as a mark-up on marginal cost, where the mark-up is given by $\theta/(\theta-1)$. Given that, the price of the firm's good relative to the consumer price index in each market can be written as:

$$\rho^D_t(z) = \frac{p^D_t(z)}{P_t} = \frac{\theta}{\theta-1}\varphi_{z,t}, \tag{15}$$

and

$$\rho^X_t(z) = \frac{p^X_t(z)}{P^*_t} = \frac{\tau}{Q_t}\rho^D_t(z), \tag{16}$$

where $\rho^D_t(z)$ is the real price of domestic goods, $\rho^X_t(z)$ is the real price of exported goods, and $Q_t = \epsilon^* \frac{P^*_t}{P_t}$, is the real exchange rate.

Total firm profits are then made up of a domestic component, d^D_t, and a component from exporting, d^X_t. Given the fixed costs of domestic production and exporting, there will be some firms that do not draw a high enough idiosyncratic productivity to make a profit (or break even) in the domestic market, and some firms that do not export. Thus, there exists cutoff productivity levels, below which a firm will not produce for either the domestic market, $z_{Dt} = \inf\{z : d^D_t \geqslant 0\}$ nor for the foreign market, $z_{Xt} = \inf\{z : d^X_t \geqslant 0\}$.

We assume that the lower bound of the productivity distribution z_{min} is low enough compared to the domestic cutoff level z_{Dt}, so that this is above z_{min}. We further assume that the domestic cutoff level z_{Dt} is low enough relative to the export cutoff level z_{Xt}, so that this is above z_{Dt}. These assumptions ensure that: 1) there will be an endogenously determined subset of firms that pay the sunk entry cost f^E, but do not then produce for the domestic market, and 2) there will exist an endogenously determined non-traded sector - those firms with productivities between z_{Dt} and z_{Xt}. These two subsets will fluctuate over time depending on the profitability of domestic production and the profitability of exporting. Firm profits are therefore as follows, with analogous equations holding for the Foreign country:

$$d_t(z) = d_t^D(z) + d_t^X(z)$$

$$d_t^D(z) = \begin{cases} \frac{1}{\theta}(\rho_t^D(z))^{1-\theta} Y_t^C - f^D, & \text{if } z \geqslant z_{Dt} \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

$$d_t^X(z) = \begin{cases} \frac{Q_t}{\theta}(\rho_t^X(z))^{1-\theta} Y_t^{C^*} - f^X, & \text{if } z \geqslant z_{Xt} \\ 0, & \text{otherwise,} \end{cases} \tag{18}$$

where Y_t^C and $Y_t^{C^*}$ denote aggregate demand in the Home and Foreign countries, respectively.

Firm Averages

In every period, there is a number of firms, N_{Dt}, that produce for the domestic market, given the cutoff level of domestic production, z_{Dt}. A number of these firms, given by N_{Xt} , export to the foreign country as well. In a similar manner to Melitz (2003), we define 'average' productivity for all domestic firms, $\tilde{z}_{Dt}$, and for firms that export to the foreign country, $\tilde{z}_{Xt}$, as:

$$\tilde{z}_{Dt} = \left[\frac{1}{1 - G(z_{Dt})} \int_{z_{Dt}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{Xt} = \left[\frac{1}{1 - G(z_{Xt})} \int_{z_{Xt}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}.$$

Melitz (2003) shows that these averages contain all the information on the productivity distributions relevant for macroeconomic variables. Therefore, our model is isomorphic to one where N_t^D firms with productivity z_D produce for the domestic market, and N_t^X firms with productivities z_X produce for the export market. Note that, if idiosyncratic firm productivities were not drawn every period, but were instead drawn on market entry and fixed thereafter as in Ghironi and Melitz (2005), then we could no longer make this assumption about the average productivity of firms that produce in the domestic market. Therefore, the average price in the domestic market, will be equal to the price of the firm with average productivity, $\tilde{p}_t^D = p_t^D(\tilde{z}_D)$, and the average price in the exporting market will be equal to the price of the exporting firms with average productivity $\tilde{p}_t^X = p_t^X(\tilde{z}_X)$. The home price index, derived from the weighted prices from domestic firms and imports from foreign firms can therefore be written as:

$$P_t = [N_{Dt}(\tilde{p}_{Dt})^{1-\theta} + N_{Xt}^*(\tilde{p}_{Xt}^*)^{1-\theta}]^{\frac{1}{1-\theta}}.$$

When combined with the price definitions in (15) and (16), it can be shown that:

$$N_{Dt}(\tilde{\rho}_{Dt})^{1-\theta} + N_{Xt}^*(\tilde{\rho}_{Xt}^*)^{1-\theta} = 1. \tag{19}$$

Average total profits are given by the sum of average profits from domestic sales and average profits from exporting, adjusted to the proportion of firms that export to a particular market, less total vacancy posting cost, and the cost of firing:

$$\tilde{d}_t = \tilde{d}_t^D + (1 - G(z_{Xt}))\tilde{d}_t^X - \kappa v_t - (\tilde{\lambda}_t l_t F_t)/(1 - \tilde{\lambda}_t).$$

This equation can then be written explicitly with the ratios of exporting firms to total domestic firms:

$$\tilde{d}_t = \tilde{d}_t^D + \frac{N_t^X}{N_t^D}\tilde{d}_t^X - \kappa v_t - (\lambda_t l_t F_t)/(1 - \lambda_t). \tag{20}$$

Firm Entry and Exit

In each period, N_t^E new firms will pay the sunk entry cost to commence production, and then find out their idiosyncratic productivity, z, at the end of the period, in the same manner as firms which are already producing. Firms will choose to enter the market until the average firm value is equal to the initial entry cost f^E, as well as the cost of posting initial vacancies, which leads to the free entry condition:

$$\tilde{vl}_t = f^E + \kappa\tilde{l}_t/q_t.$$

The number of firms operating at the end of a period, N_t^H, will be equal to the number of firms operating at the start of the period, N_t^D, plus the number of new entrants. The number of firms at the end of a period will therefore be given by $N_t^H = N_t^D + N_t^E$. At the start of a period, after shocks have hit, but before production has started, firms decide whether to produce in that period, or exit the market, depending on whether their firm-level productivity, z, is greater or lower than the cutoff level for domestic production for that period, z_{Dt}. If their productivity is higher than the domestic cutoff productivity level, $z > z_{Dt}$, they will produce, if not, they exit the market. The number of firms operating during a period will therefore be given by:

$$N_t^D = 1 - G(z_{Dt})N_{t-1}^H = 1 - G(z_{Dt})(N_{t-1}^D + N_{t-1}^E). \tag{21}$$

In the models of Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014), firms only exit the market if they are hit by an exogenous probability of death, δ. However, as in Millard *et al.* (2019), we find this a restrictive approach to modelling firm exit, since it is known that it is time invariant, and it depends on a number of factors. Therefore, in our model, we allow for an endogenously determined probability of death, $(1 - G(z_{Dt}))$, which can change from one period to the next, depending on the endogenous variation in the cutoff productivity for domestic production.

Firm Value

All producing firms, other than the firm with productivity equal to the cutoff level, $z = z_{Dt}$, make positive profits. Thus, the average profit level in the home country will be positive ($\tilde{d}_t > 0$), and the average firm will have a positive value, derived from expected future profits. At the start of a period, an endogenously determined proportion, $(1 - G(z_{Dt}))$, of firms in each country will cease to operate. Given that these firms cease to operate after new entrants have entered the market, an identical proportion of the new entrants will never operate. Since households own the firms, we can solve the households problem to calculate the average value of firms in the economy, $\tilde{v}_t$. Given that the firms are owned entirely by domestic households, the value of a firm on entry will be given by the limit of the household share Euler equation. If we assume that there are no bubbles in the economy implying $\lim_{j\to\infty} \tilde{\beta}\tilde{v}_{t+j} = 0$, where $\tilde{\beta} = \beta^j E_t \left(\frac{C^h_{t+j}}{C^h_t}\right)^{-\gamma}$, then the value of a firm will be equal to the discounted present value of its expected profit stream:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [\beta(1 - G(z_{Ds}))]^{s-1} \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} \tilde{d}_s.$$

Thus, as long as $\tilde{d}_t$ is positive, the average firm value in the home country will also be positive, $\tilde{v}_t > 0$.

Parametrising Firm Level Productivity

To solve the model it is first necessary to assume a distribution for $G(z)$. Choosing a Pareto distribution with lower bound z_{min} and shape parameter $k > \theta - 1$, it can be shown that $G(z) = 1 - (z_{min}/z)^k$. The average productivities then become $\tilde{z}_D = \nu z_{Dt}$ and $\tilde{z}_{Xt} = \nu z_{Xt}$, where $\nu = [k/(k - (\theta - 1))]^{\frac{1}{\theta-1}}$. The proportion of home firms that export is given by:

$$\frac{N^X_t}{N^D_t} = \frac{1 - G(z_{Xt})}{1 - G(z_{Dt})}.$$

Using the new definitions for *G(z)* and average productivities this can then be rewritten as:

$$\frac{N^X_t}{N^D_t} = \frac{\left(\frac{z_{min}}{z_{Xt}}\right)^k}{\left(\frac{z_{min}}{z_{Dt}}\right)^k} = (\tilde{z}_{Dt})^k (\tilde{z}_{Xit})^{-k}. \tag{22}$$

We can also rewrite the producing firm law of motion using the definition for $G(z_{Dt})$:

$$N^D_t = \left(\frac{z_{min}}{z_{Dt}}\right)^k (N^D_{t-1} + N^E_{t-1}). \tag{23}$$

Combining the zero domestic profit cutoff condition, $d^D_t(z_{Dt}) = 0$, the zero export profit cutoff condition, $d^X_t(z_{Xt}) = 0$, and the definitions for average productivity and profits, we get that the average domestic profits and the average export profits will satisfy:

$$\tilde{d}^D_t = (\theta - 1)\left(\frac{\nu^{\theta-1}}{k}\right) f^D, \tag{24}$$

$$\tilde{d}^X_t = (\theta - 1)\left(\frac{\nu^{\theta-1}}{k}\right) f^X. \tag{25}$$

Equilibrium

Total productive employment will be given by $L_t = N^D_t \tilde{l}_t$, and the total number of posted vacancies will be $V_t = N^D_t \tilde{v}_t + N_{Et}\tilde{l}_t/q_t$. Total pre-hiring unemployment is given by:

$$U_t = L^F - (1 - \lambda^x)\left(\frac{z_{min}}{z_{Dt}}\right)^k \tilde{l}_{t-1}(N^D_{t-1} + N^E_{t-1}).$$

Aggregating the firms production function (4) across all producing and exporting firms the aggregate production function is obtained:

$$Z_t \tilde{a}_t L_t = (\tilde{\rho}_t^D)^{-\theta} Y_t^C (\tilde{z}_{Dt})^{-1} N_t^D + \tau(\tilde{\rho}_t^X)^{-\theta} Y_t^{C^*} (\tilde{z}_{Xt})^{-1} N_t^X, \tag{26}$$

where, Y_t^C, aggregate demand, is given by $Y_t^C = C_t + N_t^E f^E + N_t^D f^D + N_t^X f^X + \kappa V_t + \lambda_t L_t F/(1-\lambda_t)$. Bonds are in zero net supply worldwide, so $B_t = B_t^*$. Finally, the change in the net foreign assets of a country between period t and $t+1$ will be determined by that countries current account balance:

$$B_{t+1} - B_t - Q_t(B_{t+1}^* - B_t^*) = CA_t \equiv -r_{t-1} B_t + Q_t r_{t-1}^* B_t + TB_t, \tag{27}$$

where $TB_t \equiv Q_t N_t^X (\tilde{\rho}_t^X)^{1-\theta} Y_t^{C^*} - N_t^{X^*} (\tilde{\rho}_t^{X^*})^{1-\theta} C_t$, is the trade balance, defined as the difference between total exports and total imports, expressed in units of Home country consumption good.

3 Calibration

In our calibration, full symmetry is assumed between the Home and Foreign countries. Periods are interpreted as quarters, and the discount factor, β, and the risk aversion parameter, γ, are thus set to 0.99 and 2, respectively, both of which are standard values in quarterly business cycle models. The quadratic bond holding cost, ξ, is set to 0.0025 as in Ghironi and Melitz (2005) and Cacciatore (2014), to ensure stationarity but not overstate the role of these costs in driving the responses of the endogenous variables to macroeconomic shocks. The elasticity of substitution across product varieties, θ, is obtained from Bernard *et al.* (2003) and set to 3.8.[2] The value for the shape parameter, k, is also obtained from Bernard *et al.* (2003) and is set to 3.4 (also satisfying the condition that $k > \theta - 1$). The per unit iceberg costs τ were set to 1.22, to match the iceberg cost for trade between the USA and Canada in the *World Bank Trade Costs* database. The fixed costs of exporting, f^X, are set such that the proportion of firms that export is 1%, as is the case for the USA, according to the *California Inland Empire District Export Council*. The fixed costs of domestic production are set such that the endogenous firm exit rate, $\left(\frac{z_{Dt}}{z_{min}}\right)^k$, matches the $2.9\overline{6}$ pre-Great Recession USA quarterly firm death rate, according to date from the *US Bureau of Labor Statistics*.[3] The sunk entry cost, f_E, is set to 5.2 months of per capita output, as in Ebell and Haefke (2009), and the labour forces, L^F, are normalised to 1, as in Ghironi and Melitz (2005), Cacciatore (2014) and Millard *et al.* (2019).

The unemployment elasticity of the matching function, ε, is set to 0.4, consistent with Blanchard and Diamond (1991), the bargaining power, η, is set to 0.4, as estimated in Flinn (2006). The unemployment benefit u^b is set such as the replacement rate, $u^b/\tilde{w}$, matches the replacement rate reported by the *OECD Benefits and Wages* statistics (OECD (2004)). This corresponds to a 24% replacement rate in the USA. As in Cacciatore (2014), the firing costs, F, are set to 0.15 times the average wage. The value of λ^x is set so that, as in den Haan *et al.* (2000), exogenous separation accounts for 68% of within firm separations.

The remaining labour market parameters are calibrated as follows: the cost of posting a vacancy, κ, the disutility from work, υ, and the matching efficiency, χ, are calibrated to jointly match the steady state

[2] It is important to note that, although the value of θ may appear low (standard macro literature sets θ to 6 to deliver a 20% mark-up over marginal cost) the mark-up in this paper represents mark-up over average cost, including the entry cost. Thus although $\theta = 3.8$ implies a high mark-up over marginal cost, the markup over average cost is reasonable. We run the same simulations for values of θ of 5 and 6, which deliver similar results to $\theta = 3.8$.

[3] We have experimented with different values for the fixed costs of domestic production and we have found that the responses presented were very similar in all cases, as long as the ratio of fixed costs of exporting to fixed costs of domestic production was maintained, through the targeting of the proportion of firms that export.

job finding probability, probability of filling a vacancy, and pre-Great Recession USA Unemployment Rate. The probability that an unemployed worker will meet a match, ι, is set to 0.75 as estimated in Hobijn and Sahin (2007), and the probability of a vacancy posted by a firm resulting in a match, q, is set to 0.9, following Andolfatto (1996). The unemployment rate is set to 5.2% to match *US Bureau of Labor Statistics* data. Finally, the job specific productivity shock, μ, is set to 0, and σ is calibrated to replicated the volatility of employment relative to GDP, as in den Haan *et al.* (2000). Idiosyncratic productivity innovations a are lognormally distributed with c.d.f. $H(a; \mu_a, \delta_a)$.

4 Transmission of Macroeconomic Shocks

In this section, we examine the extent to which the interactions between the entry and exit of less productive non-trading firms into and out of the domestic market and labour market frictions impact the transmission of macroeconomic shocks. Therefore, more specifically, we examine the response of transitory shocks to aggregate technology, sunk entry costs and the fixed costs of domestic production on labour productivity, and quantify the extent to which each of them impact labour productivity persistence. We show that the interactions between entry and exit of less-productive non-trading domestic firms and labour market frictions, introduce additional endogeneity into the persistence of productivity, depending on the source of the shock.

For the purpose of our analysis, we adopt the definition for aggregate labour productivity specified in Kehrig (2015). Productivity is thus defined as GDP divided by the sum of labour used in production, L_{Pt}, and labour used to pay the fixed costs, F:

$$Z_{Ot} = \frac{GDP_t}{L_{Pt} + F},$$

where $GDP_t = L_t w_t + N_{Dt}\tilde{d}_t - v_t N_{Et}$, is GDP defined by the income approach, which is equal to GDP defined by the output and expenditure approaches (but easier to define). The labour used to pay the fixed costs, F, is equal to the sum of the labour used to pay the fixed costs of domestic production, $\frac{N_{Dt} f_{Dt}}{Z_t}$, and the fixed costs of exporting, $\frac{N_{Xt} f_{Xt}}{Z_t^h}$, to the foreign country. All future references to labour productivity should be taken to refer to productivity as measured above.

Figure 1 shows the response of labour productivity in the Home country to three macroeconomic shocks: a one period, 0.9753 percentage point positive shock to aggregate technology, Z, shown by the dashed line; a one period, 13.017 percentage point negative shock to the sunk costs of entry, f^E, shown by the dotted line; and a one period, 3.118 percentage point negative shock to the fixed costs of domestic production, f^D, shown by the dot-dash line. The solid line shows the time path of the shocks themselves and the length of time after the shock (in quarters) is on the horizontal axis.

We assume that these shocks hit at the beginning of the period, before production starts, and follow an AR(1) processes:

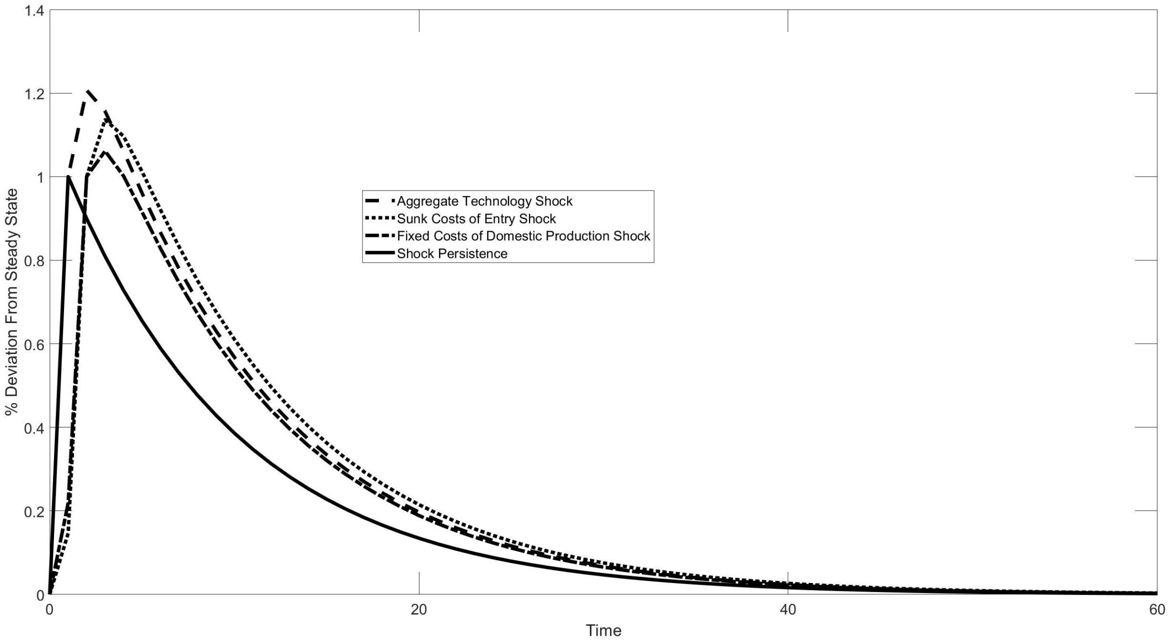
$$\hat{Z}_t = \rho \hat{Z}_{t-1} + \epsilon_t,$$

$$\hat{f_t^E} = \rho \hat{f^E}_{t-1} + \epsilon_t,$$

$$\hat{f_t^D} = \rho \hat{f^D}_{t-1} + \epsilon_t,$$

where $\hat{Z}_t$, $\hat{f_t^E}$ and $\hat{f_t^D}$ are the deviations of aggregate technology, sunk costs of entry and fixed costs of domestic production, respectively, from their steady-state levels in period t. ρ is the exogenously set persistence of shocks, and ϵ_t is the magnitude of the shock in period t. We assume that the persistence of the shocks, exogenously given, is equal to $\rho = 0.90$, as in Ghironi and Melitz (2005).[4]

Figure 1: Response of Labour Productivity to Macroeconomic Shocks



If the number of firms and the distributions of firm productivities were fixed exogenously as in Millard *et al.* (2019)/ Ghironi and Melitz (2005), then the persistence of labour productivity would have been solely determined by the exogenously given persistence parameter for the macroeconomic shocks, ρ, as is the case in standard RBC models. However, when the distribution of firm productivities is endogenously determined through endogenous changes in the cut-off productivity levels, an endogenous degree of persistence is now present in the response of labour productivity to each of these shocks.

The interactions between the entry and exit of non-trading firms and labour market frictions introduces additional endogeneity into the persistence of productivity, through the endogenous movement of labour into and out of employment, and the degree of persistence depends, depends on the source of the macroeconomic shock:

1. An aggregate technology shock, Z, induces labour productivity, Z_O, to return to its steady-state level at a much slower rate than aggregate technology, due to the behaviour of the average firm-level productivity, z_D, the job specific productivity, $\tilde{a}$, and the average labour employed per firm, $\tilde{l}$. Firm-level productivity and job-specific productivity both remain above their steady-state levels for a significantly longer period of time than aggregate technology, and average labour employed per firm remains below its steady state for a similar period of time. The half-life of productivity in response to the aggregate technology shock is 12 periods, compared to a half-life of 8 periods for the shock itself. The overall persistence of labour productivity is 0.943, compared to a shock persistence of 0.9. Comparing the behaviour of labour productivity in response to a shock to aggregate technology in our model, to that in a model where there are no labour market frictions (as in Millard *et al.* (2019)), the impact of the interactions between firm entry and exit and labour

[4]We conduct robustness checks on the value of ρ, examining the responses of labour productivity for a range of values from $\rho = 0.85$, as in Pancrazi and Vukotic (2011) to $\rho = 0.994$, as in Baxter (1995). For this range of values, the responses were similar in all cases. Results are available on request.

market frictions is limited. Both the half life of the shock, and the persistence equivalent are identical.

2. A shock to the sunk costs of entry, f^E, induces labour productivity, Z_O, to return to its steady state at a much slower rate than both the sunk cost of entry shock and productivity in the case of a shock to aggregate technology, due to the behaviour of the average firm-level productivity, z_D, and the average labour employed per firm, $\tilde{l}$. Firm-level productivity remains above its steady-state level while average labour employed per firm remains below its steady state for a significantly longer time than the sunk costs of entry. The half-life of productivity in response to the sunk costs of entry shock is 12 periods, compared to a half-life of 8 periods for the shock itself. The overall persistence of labour productivity is 0.952, compared to a shock persistence of 0.9. Clearly, the interactions between non-trading firm entry and exit and labour market frictions have introduced further persistence into the response of productivity above and beyond that seen in a model with only non-trading firm entry and exit and without labour market frictions (Millard *et al.* (2019)). In Millard *et al.* (2019), the absence of such interactions, intertemporal bond smoothing results in almost no increase in persistence, thus the persistence of the response of productivity is equivalent to the persistence of the shock.

3. A shock to the fixed costs of domestic production, f_D, induces labour productivity, Z_O, to return to its steady state at a much slower rate than both the fixed costs of domestic production shock and productivity seen in the case of a shock to aggregate technology. This is due to the behaviour of the average labour employed per firm, $\tilde{l}$, and the number of firms, N_D. Average labour employed per firm remains below its steady state, and the number of firms remains above the steady state for a significantly longer time than the fixed costs of domestic production. Again, the half-life of productivity in response to the sunk costs of entry shock is 11 periods, compared to a half-life of 8 periods for the shock itself. The overall persistence of labour productivity is 0.954 for productivity, compared to a shock persistence of 0.9.

   Comparing these results to Millard *et al.* (2019), we see that the interactions between non-trading firm entry and exit and labour market frictions has introduced significant persistence into the response of productivity. In the absence of such interactions, given that average firm productivity returns to steady state quickly, productivity falls much more quickly than the shock itself.

The wide variation in empirically calculated figures for the persistence of labour productivity seen in the literature, which ranges from 0.85 in Pancrazi and Vukotic (2011) to 0.906 in Backus *et al.* (1992) and 0.994 in Baxter (1995), could be explained in our framework, by the source of the macroeconomic shock. The persistence of labour productivity across the various macroeconomic shocks studied here is driven by the responses of firms and consumers, which differ in each case. Therefore, in the next section we are going to present the dynamic response of the main macroeconomic variables to each of these transitory shocks.

In Figure 2 we show the dynamic response of consumption, C, average productivity for domestic production, z_D, labour productivity, Z_O, the real wage, w, the number of firms, N_D, the number of new entrants, N_E, the job-specific productivity, $\tilde{a}$, the unemployment rate and the average labour employed per firm , $\tilde{l}$, to the one period transitory shocks outlined above, to aggregate technology, the sunk costs of entry and the fixed costs of domestic production. The solid lines plot the response to the aggregate technology shock, the dotted lines plot the response to the sunk cost of entry shock, and the dashed lines plot the response to the fixed costs of domestic production shock.

Figure 2: Comparison of Response to Macroeconomic Shocks

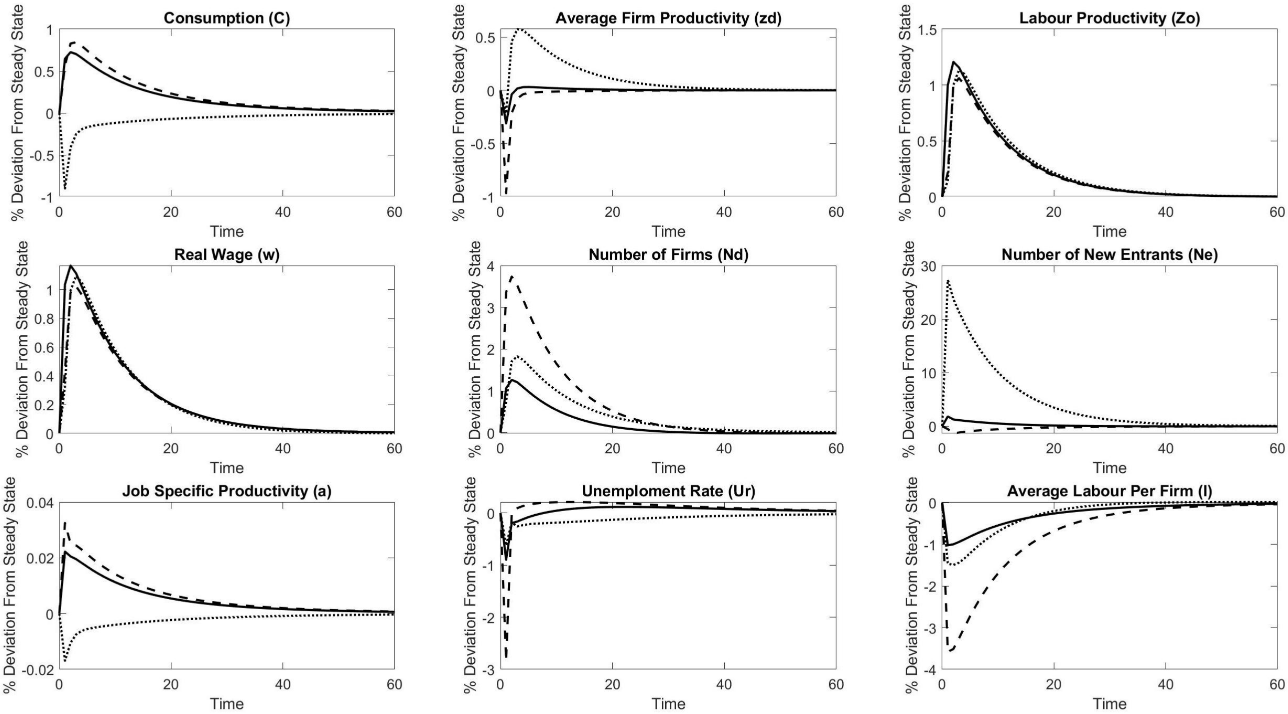


Figure 2 shows that, in response to a positive aggregate technology shock firms with lower firm-level productivity, who would otherwise be unable to enter the market, now enter the market, decreasing in the first instance average domestic firm-level productivity, z_D. However, as relatively less productive firms now remain in the market, (instead of endogenously exiting) and new firms enter the market, N_E, driven by potentially higher profits, the relatively less productive firms are forced out of the market, due to increased competition in the market and as a result average domestic firm-level productivity then increases. The increase in the number of firms, N_D, increases the demand for labour, leading to increases in the real wage, w, and decreases in the average labour employed per firm, $\tilde{l}$, as firms retain only their most productive worker matches. Given that the number of firms and the labour demand returns to trend as the shock dissipates, the only force keeping productivity above trend is the average firm productivity. Therefore, the persistence of productivity in a model with both endogenous entry and exit and labour market frictions is identical to that in a model with only endogenous entry and exit, i.e. Millard *et al.* (2019).

Comparing our results for the path of consumption, C, to the relevant results in Ghironi and Melitz (2005), Cacciatore (2014) and Millard *et al.* (2019), we can see that the interactions between the entry and exit of non-trading firms and labour market frictions amplifies the impact of shocks to technology to a greater extent than either labour market frictions or endogenous entry and exit can individually. In our model, consumption peaks at 0.725% above steady state, compared to 0.35% in the baseline model of Ghironi and Melitz (2005) , 0.38% percent in the model with labour market frictions only of Cacciatore (2014), and 0.6% in the model with endogenous entry and exit only of Millard *et al.* (2019). The larger consumption increase is driven by the higher average firm productivity, and also by the higher in job-specific productivity and the reduced unemployment. More workers are employed (and therefore more workers are receiving a wage), and the increased productivity of the worker-firm match increases the wages for the already employer workers even more/further.

In response to an aggregate technology shock, the unemployment rate falls by more in pur model, to almost -1% below the steady state, comparing to just under 0.6% in Cacciatore (2014). In our model we have a higher unemployment rate as endogenous firm entry and exit introduces additional friction on the labour market, and these are well captured in out model.

In response to a negative shock to the sunk costs of entry, as Figure 2 shows, average firm level productivity, z_D, responds to a much greater extent than in response to a shock to aggregate technology. The number of new entrants,N^E, increases dramatically, resulting in larger increases in the number of firms, N_D. As new firms enter the market, the demand for labour increases, resulting in increases in the real wage, w, which leads to decreases in the average labour employed per firm, $\tilde{l}$, as firms can only afford to retain a smaller number of workers on average. Given that the increase in the number of firms is larger than the decrease in the average labour per firm, unemployment also decreases. Since a greater proportion of aggregate demand is now needed to pay the fixed costs of domestic production and the sunk cost of entering the market, as a result of the larger number of firms and new entrants respectively, consumption decreases. As the sunk cost of entry shock dissipates, the number of new entrants returns to trend, as does the number of firms, average labour employed per firm and average firm productivity, although the number of firms, average labour employed and average productivity return to trend more slowly, as a result of the persistence in the firm law of motion and the employment law of motion. Aggregate productivity therefore also falls back to trend more slowly than the shock, resulting in a degree of endogeneity in the persistence of productivity. The ability of firms to lower their average labour employment amplifies the response of the average real wage and productivity, as the profitability and productivity of the remaining matches between workers and firms are higher, which is not captured in Millard *et al.* (2019), since they are missing in their model labour markets frictions.

Figure 2 also shows that, in response to a negative shock to the fixed costs of domestic production, the cutoff productivity for domestic production falls, resulting in a fall in average firm productivity, z_D. The reductions in cutoff productivity also allows many firms who would have exited the market to now remain in the market, leading to increases in the number of firms, N_D. As in the case of the sunk costs of entry shock, the increase in the number of firms is associated with an increase in the demand for labour, resulting in a higher real wage, w, which leads to a decrease in the average labour employed per firm, $\tilde{l}$, as firms can only afford to retain a smaller number of workers on average. As firms are now only retaining their most productive matches, this results in an increase in labour productivity. Once the number of firms has increased and the wage increased to match, the cutoff productivity for domestic production returns quickly to steady state, as a result of the increased competition. The increase in the number of firms and decrease in the average labour employed per firm however, remain persistently above trend, causing labour productivity to also remain persistently above trend, for much longer than the duration of the shock itself. In models without an endogenous response of labour such as Millard *et al.* (2019),productivity returns quickly to trend, as average firm level productivity also returns to trend.

In this paper we have examined the response of productivity and other key macroeconomic variables to transitory shocks to aggregate technology, the sunk costs of entry and the fixed costs of domestic production. We have show that in response to the shocks to the sunk costs of entry and the fixed costs of domestic production, the interactions between endogenous entry and exit and labour market frictions results in a greater degree of endogenous persistence in the response of productivity, compared to models such as Millard *et al.* (2019) that only allow for endogenous entry and exit. We have also shown that the interactions amplify the impact of a shock to aggregate technology on consumption, compared to both a model with only endogenous entry and exit (Millard *et al.* (2019)), but also a model with only labour market frictions (Cacciatore (2014)). As inGhironi and Melitz (2005), our model too generates varying degrees of persistence of productivity changes in responses to shocks, depending on the source/origin/nature of the shock, which could provide an explanation for the wide variation in the empirically calculated figures for the persistence of productivity seen in the literature, ranging from 0.85 in Pancrazi and Vukotic (2011) to 0.906 in Backus *et al.* (1992) and 0.994 in Baxter (1995).

5 Conclusion

In this paper we have developed a dynamic, stochastic, general equilibrium (DSGE) model of international trade with monopolistic competition, Mortensen-Pissarades labour market frictions and heterogeneous firms, in order to study the impact of the interactions between labour market frictions and the entry and exit of less-productive non-trading firms into and out of the domestic market on the response of labour productivity and unemployment to various macroeconomic shocks.

The main result standing out from our study is that the endogenous entry and exit of non-trading firms into and out of the domestic market to create endogenous fluctuations in vacancy posting and unemployment, generates further endogeneity into the persistence of productivity. This is above and beyond the persistence generated solely by non-trading firm entry and exit, as in Millard *et al.* (2019), or solely by labour market frictions, as in Cacciatore (2014), showing that labour market rigidities do affect productivity through their impact on the less productive non-trading firms and making labour productivity more persistent. Furthermore, shocks to the sunk costs of entry and the fixed costs of domestic production now generate much greater persistence in the response of productivity to the macroeconomic shocks. We therefore, provide a framework suitable to analyse the drivers of the wide variation in the empirically calculated figures for the persistence of productivity ranging from 0.85 in Pancrazi and Vukotic (2011) to 0.906 in Backus *et al.* (1992) and 0.994 in Baxter (1995).

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